The RAM Model and Runtime Analysis

COMS10017 - (Object-Oriented Programming and) Algorithms

Dr Christian Konrad

What is an Algorithm?

Muhammad ibn Musa **al-Khwarizmi**

$\sim$ 780 - $\sim$ 850

($\approx$ Algorithms)

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- Computational procedure to solve a computational problem

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- Which individual steps can an algorithm do?
  Depends on computer, programming language, ...
- How long do these steps take?
  Depends on computer, compiler optimization, ...

Models of Computation

Real Computers are complicated

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Memory hierachy

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Memory hierachy, floating point operations

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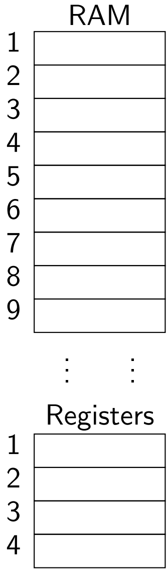
See also:

COMS20007: Programming Languages and Computation

RAM Model

RAM Model: Random Access Machine Model



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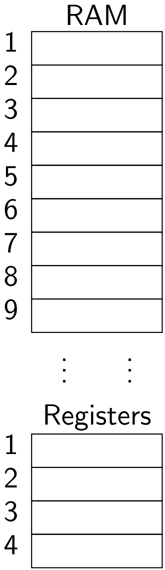

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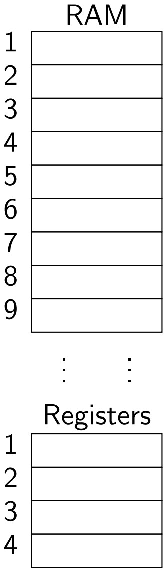

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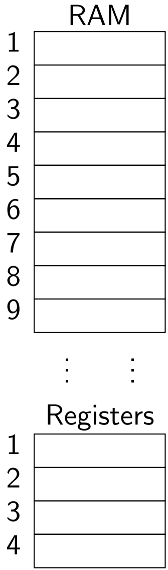

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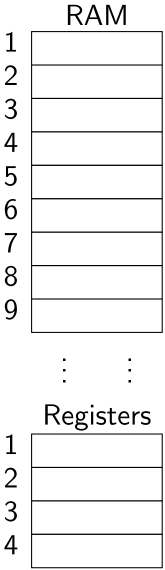

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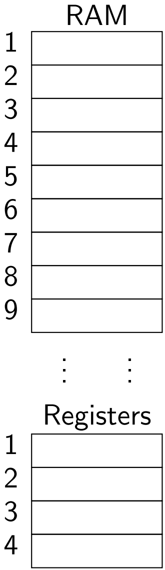

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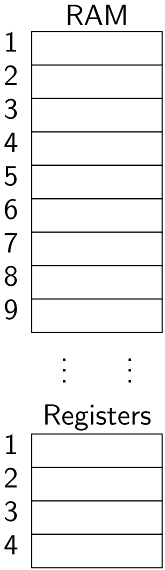

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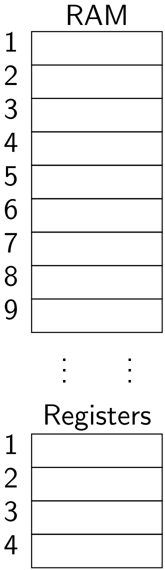

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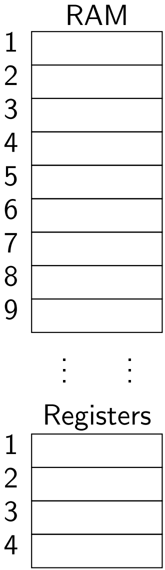

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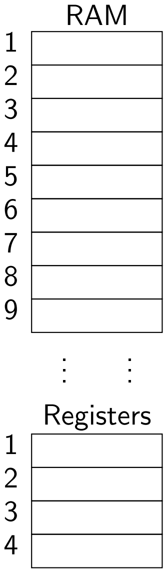

In a single Time Step we can:

- Load a word from memory into a register
- Compute $(+, -, *, /)$, bit operations, comparisons, etc. on registers
- Move a word from register to memory

RAM Model (2)

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Sequence of elementary operations (similar to assembler code)

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- Assume that $M[0]$ and $M[1]$ contain the integers
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- This space is not accounted for

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How to specify an Algorithm

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Exercise: How to implement in RAM model?

Require: Array of n integers A
  $S \leftarrow 0$
  for $i = 0, \dots, n-1$ **do**
    $S \leftarrow S + A[i]$
  return S

Notions of Runtime

Runtime on Specific Input
Given a specific input X, what is the number of elementary operations of the algorithm on X?

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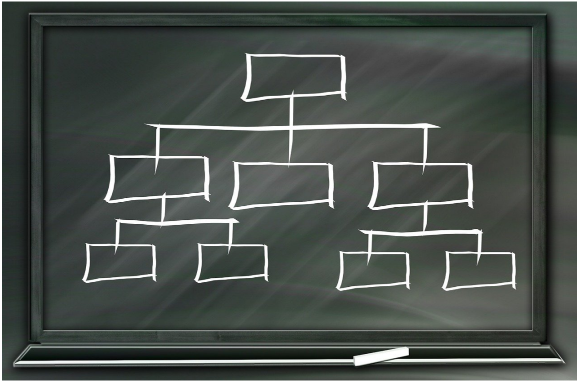
Best-case
Consider the set of all inputs of length n. What is the minimum number of elementary operations executed by the algorithm when run on every input of this set?

Average-case
Consider a set of inputs (e.g. the set of all inputs of length n). What is the average number of elementary operations executed by the algorithm when run on every input of this set?

Hierachy

Runtime Hierachy:



$$\textbf{Best-case} = O(\textbf{Average-case}) = O(\textbf{Worst-case})$$

Runtime/Space Analysis of Algorithms

Runtime

Goals:

Runtime

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- **Runtime:** Count number of elementary operations when implemented in RAM model

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Solution:

- Analyze algorithm as specified in pseudo code directly

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However...

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- We would like to state and analyze our algorithms in pseudo code (or a programming language, natural language, . . .)

Solution:

- Analyze algorithm as specified in pseudo code directly
- Make sure that every instruction can be implemented in the RAM model using $O(1)$ elementary operations

Example

Require: Integer array A of length n

$s \leftarrow 0$
for $i \leftarrow 0 \ldots n-1$ **do**
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$s \leftarrow 0$ $\qquad$ O(1)

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Example

Require: Integer array A of length n

$s \leftarrow 0$	O(1)
for $i \leftarrow 0 \dots n-1$ **do**	n **times**
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Runtime: $O(1) + n \cdot O(1) + O(1) =$

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Example 2

Require: Integer array A of length n

```
s ← 0
for i ← 0 ... n − 1 do
    for j ← i ... 2i do
        s ← s + A[i]
return s
```

Example 2

Require: Integer array A of length n

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Require: Integer array A of length n
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  for i ← 0 ... n − 1 do
    for j ← i ... 2i do                   i + 1 times
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  return s                                O(1)
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Runtime:

$$O(1) + \sum_{i=0}^{n-1} ((i+1) \cdot O(1)) + O(1)$$

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$$= O(1) + O(1) \sum_{i=1}^{n} i$$

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$$= O(1) + O(1)\sum_{i=1}^{n} i = O(1) + O(1)\frac{n(n+1)}{2}$$

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$$= O(1) + O(1)\sum_{i=1}^{n} i = O(1) + O(1)\frac{n(n+1)}{2}$$

$$= O(1) + O(\frac{n^2}{2} + \frac{n}{2})$$

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return s	O(1)

Runtime:

$$O(1) + \sum_{i=0}^{n-1} ((i+1) \cdot O(1)) + O(1) = O(1) + O(1) \sum_{i=0}^{n-1} (i+1)$$

$$= O(1) + O(1) \sum_{i=1}^{n} i = O(1) + O(1) \frac{n(n+1)}{2}$$

$$= O(1) + O(\frac{n^2}{2} + \frac{n}{2}) = O(1) + O(n^2)$$

Example 2

Require: Integer array A of length n

$s \leftarrow 0$	O(1)
for $i \leftarrow 0 \dots n-1$ **do**	n **times**
for $j \leftarrow i \dots 2i$ **do**	$\mathrm{i}+1$ **times**
$s \leftarrow s + A[i]$	O(1)
return s	O(1)

Runtime:

$$O(1) + \sum_{i=0}^{n-1}((i+1)\cdot O(1)) + O(1) = O(1) + O(1)\sum_{i=0}^{n-1}(i+1)$$

$$= O(1) + O(1)\sum_{i=1}^{n} i = O(1) + O(1)\frac{n(n+1)}{2}$$

$$= O(1) + O(\frac{n^2}{2} + \frac{n}{2}) = O(1) + O(n^2) = O(n^2)\ .$$

Example 3

Algorithm: Given is an integer array of length n. Run through the array from left to right and maintain the minimum seen so far.

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Runtime: $O(n)$