Dynamic Programming - Pole Cutting

COMS10017 - (Object-Oriented Programming and) Algorithms

Dr Christian Konrad

Pole Cutting

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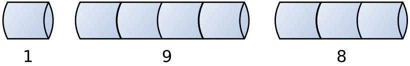

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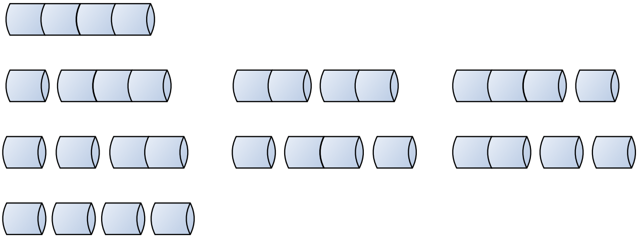

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- Find best out of 2^{n-1} possibilities
- We could disregard similar cuts, but we would still have an exponential number of possibilities
- A fast algorithm cannot try out all possibilities

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- Suppose the optimal cut uses k pieces

$$n = i_1 + i_2 + \cdots + i_k$$

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- Suppose the optimal cut uses k pieces

$$n = i_1 + i_2 + \dots + i_k$$

- Optimal revenue r_n:

$$r_n = p(i_1) + p(i_2) + \dots + p(i_k)$$

Pole Cutting (5)

What are the optimal revenues r_i?

length i	1	2	3	4	5	6	7	8	9	10
price $p(i)$	1	5	8	9	10	17	17	20	24	30

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$$r_n = \max\{p_n, r_1 + r_{n-1}, r_2 + r_{n-2}, \dots, r_{n-1} + r_1\}$$

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- $r_i + r_{n-i}$: initial cut into two pieces of sizes i and $n - i$

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Example: $n = 4$

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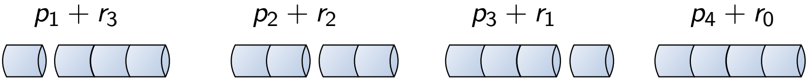

Recursive Top-down Implementation

Recall:

$$r_n = \max_{1 \le i \le n} (p_i + r_{n-i}) \text{ and } r_0 = 0 \ .$$

Direct Implementation:

```
Require: Integer n, Array p of length n with prices
  if n = 0 then
    return 0
  q ← −∞
  for i = 1 ... n do
    q ← max{q, p[i] + Cut-Pole(p, n − i)}
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Algorithm $\textsc{Cut-Pole}(p, n)$

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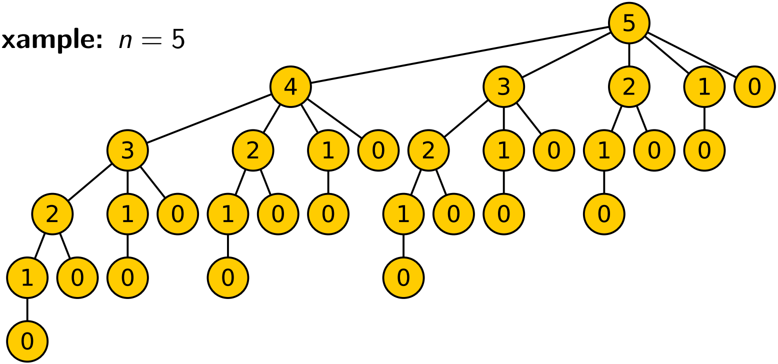
Algorithm CUT-POLE(p, n)

How efficient is this algorithm?

Recursion Tree for Cut-Pole

Example: $n = 5$



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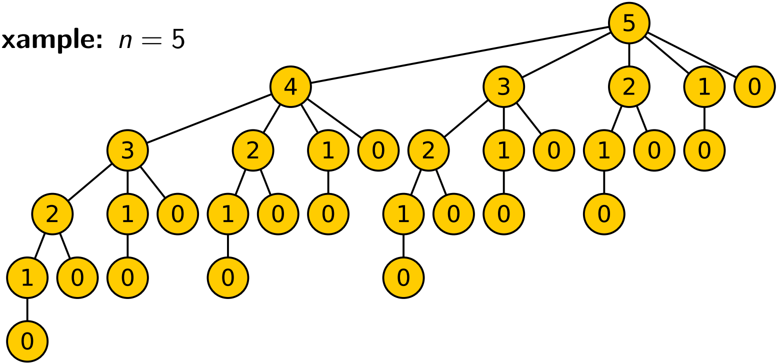

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This implies $T(i) = 2^i$.

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What can we do better?

- Observe: We compute solutions to subproblems many times
- Avoid this by storing solutions to subproblems in a table!
- This is a key feature of dynamic programming

Implementing the Dynamic Programming Approach

Top-down with memoization

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- Fill table T from smallest to largest index

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- When computing r_i, store r_i in a table T (of size n)
- Before computing r_i again, check in T whether r_i has previously been computed

Bottom-up

- Fill table T from smallest to largest index
- No recursive calls are needed for this

Top-down Approach

Require: Integer n, Array p of length n with prices
    Let $r[0 \dots n]$ be a new array
    for $i = 0 \dots n$ **do**
        $r[i] \leftarrow -\infty$
    return MEMOIZED-CUT-POLE-AUX(p, n, r)

Algorithm MEMOIZED-CUT-POLE$(p,\ n)$

- Prepare a table r of size n
- Initialize all elements of r with $-\infty$
- Actual work is done in MEMOIZED-CUT-POLE-AUX, table r is passed on to MEMOIZED-CUT-POLE-AUX

Top-down Approach (2)

Require: Integer n, array p of length n with prices, array r of revenues

```
if r[n] ≥ 0 then
   return r[n]
if n = 0 then
   q ← 0
else
   q ← −∞
   for i = 1 ... n do
      q ← max{q, p[i] + MEMOIZED-CUT-POLE-AUX(p, n − i, r)}
r[n] ← q
return q
```

Algorithm MEMOIZED-CUT-POLE-AUX(p, n, r)

Observe: If $r[n] \geq 0$ then $r[n]$ has been computed previously

Bottom-up Approach

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   for j = 1 ... n do
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      for i = 1 ... j do
         q ← max{q, p[i] + r[j − i]}
      r[j] ← q
   return r[n]
```

Algorithm BOTTOM-UP-CUT-POLE(p, n)

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Algorithm $\textsc{Bottom-Up-Cut-Pole}(p,\ n)$

Runtime: Two nested for-loops

Bottom-up Approach

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    for i = 1 ... j do
        q ← max{q, p[i] + r[j − i]}
    r[j] ← q
return r[n]
```

Algorithm BOTTOM-UP-CUT-POLE(p, n)

Runtime: Two nested for-loops

$$\sum_{j=1}^{n}\sum_{i=1}^{j} O(1) = O(1)\sum_{j=1}^{n}\sum_{i=1}^{j} 1 = O(1)\sum_{j=1}^{n} j = O(1)\frac{n(n+1)}{2} = O(n^2) \ .$$

Conclusion

Runtime of Top-down Approach $O(n^2)$

(please think about this!)

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Dynamic Programming

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- If a problem exhibits *optimal substructure* then dynamic programming is often the right choice
- Top-down and bottom-up approaches have the same runtime