The RAM Model and Runtime Analysis

COMS10017 - Algorithms 1

Dr Christian Konrad

What is an Algorithm?

Muhammad ibn Musa **al-Khwarizmi**
$\sim$ 780 - $\sim$ 850
($\approx$ Algorithms)

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- Computational procedure to solve a computational problem

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  Depends on computer, programming language, . . .
- How long do these steps take?
  Depends on computer, compiler optimization, . . .

Models of Computation

Real Computers are complicated

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Memory hierachy

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Memory hierachy, floating point operations

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Memory hierachy, floating point operations, garbage collector

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Memory hierachy, floating point operations, garbage collector, how long does x^y take?

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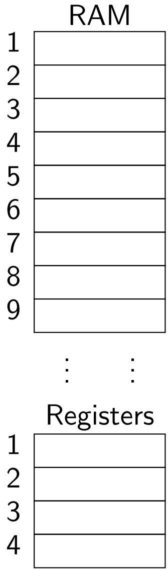
See also:

COMS20007: Programming Languages and Computation

RAM Model

RAM Model: Random Access Machine Model



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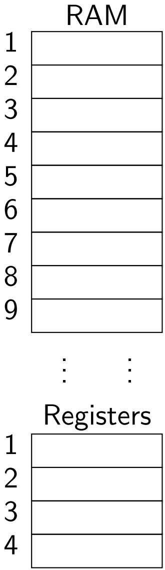

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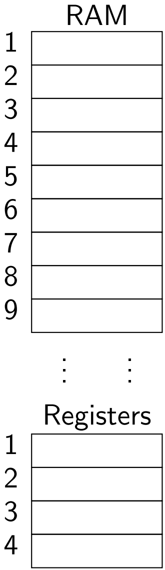

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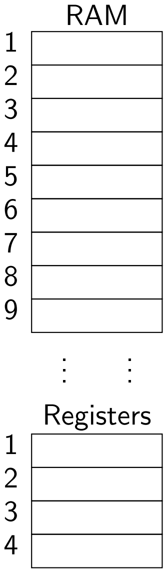

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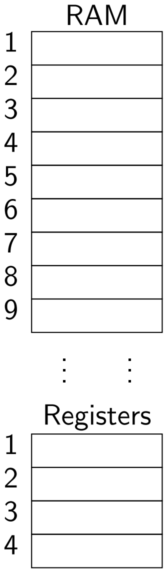

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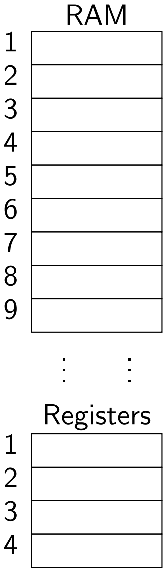

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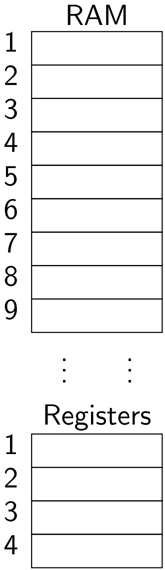

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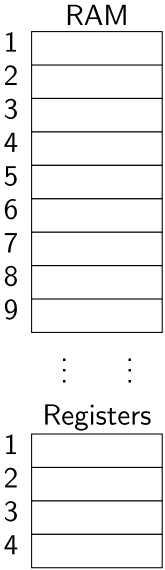

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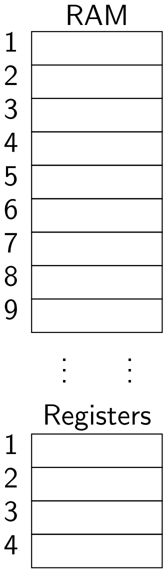

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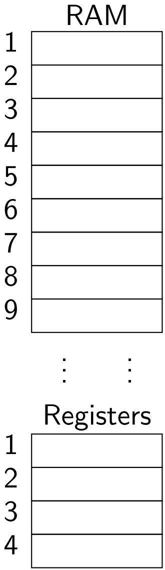

In a single Time Step we can:

- Load a word from memory into a register
- Compute $(+, -, *, /)$, bit operations, comparisons, etc. on registers
- Move a word from register to memory

RAM Model (2)

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- Assume that $M[0]$ and $M[1]$ contain the integers
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- This space is not accounted for

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Exercise: How to implement in RAM model?

```
Require: Array of n integers A
  S ← 0
  for i = 0, ..., n − 1 do
    S ← S + A[i]
  return S
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Notions of Runtime

Runtime on Specific Input
Given a specific input X, what is the number of elementary operations of the algorithm on X?

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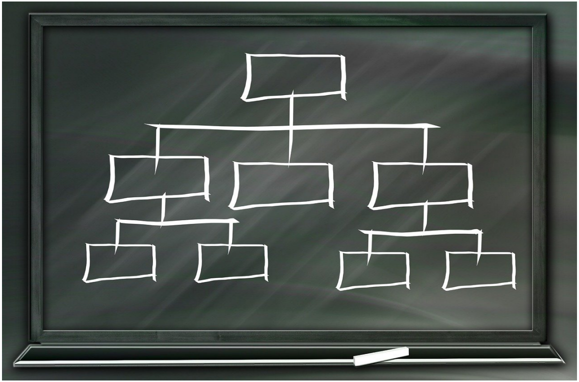
Average-case
Consider a set of inputs (e.g. the set of all inputs of length n). What is the average number of elementary operations executed by the algorithm when run on every input of this set?

Hierachy

Runtime Hierachy:



$$\textbf{Best-case} = O(\textbf{Average-case}) = O(\textbf{Worst-case})$$

Runtime/Space Analysis of Algorithms

Runtime

Goals:

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- **Runtime:** Count number of elementary operations when implemented in RAM model

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Solution:

- Analyze algorithm as specified in pseudo code directly

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- We would like to state and analyze our algorithms in pseudo code (or a programming language, natural language, . . .)

Solution:

- Analyze algorithm as specified in pseudo code directly
- Make sure that every instruction can be implemented in the RAM model using $O(1)$ elementary operations

Example

Require: Integer array A of length n

$s \leftarrow 0$
for $i \leftarrow 0 \ldots n-1$ **do**
  $s \leftarrow s + A[i]$
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s ← 0                          O(1)
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for $i \leftarrow 0 \dots n-1$ **do**	**n times**
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$$= O(1) + O(1) \sum_{i=1}^{n} i$$

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$$= O(1) + O(1)\sum_{i=1}^{n} i = O(1) + O(1)\frac{n(n+1)}{2}$$

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$$= O(1) + O(1)\sum_{i=1}^{n} i = O(1) + O(1)\frac{n(n+1)}{2}$$

$$= O(1) + O(\frac{n^2}{2} + \frac{n}{2})$$

Example 2

Require: Integer array A of length n

$s \leftarrow 0$	$\mathbf{O(1)}$
for $i \leftarrow 0 \ldots n-1$ **do**	**n times**
for $j \leftarrow i \ldots 2i$ **do**	$\mathbf{i+1}$ **times**
$s \leftarrow s + A[i]$	$\mathbf{O(1)}$
return s	$\mathbf{O(1)}$

Runtime:

$$O(1) + \sum_{i=0}^{n-1} ((i+1) \cdot O(1)) + O(1) = O(1) + O(1) \sum_{i=0}^{n-1} (i+1)$$

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