Dynamic Programming - Pole Cutting

COMS10017 - Algorithms 1

Dr Christian Konrad

Pole Cutting

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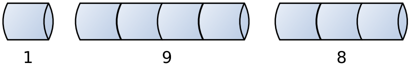

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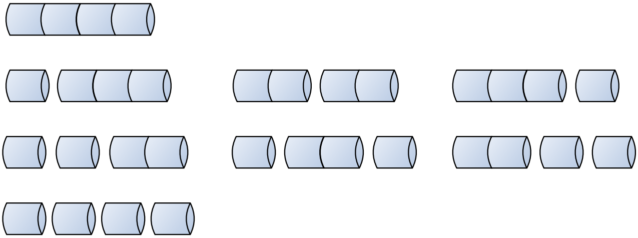

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Problem:

- Find best out of 2^{n-1} possibilities
- We could disregard similar cuts, but we would still have an exponential number of possibilities
- A fast algorithm cannot try out all possibilities

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- Suppose the optimal cut uses k pieces

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- Optimal revenue r_n:

$$r_n = p(i_1) + p(i_2) + \cdots + p(i_k)$$

Pole Cutting (5)

What are the optimal revenues r_i?

length i	1	2	3	4	5	6	7	8	9	10
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$$r_1 =$$

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\begin{aligned}
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- $r_i + r_{n-i}$: initial cut into two pieces of sizes i and $n - i$

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Example: $n = 4$

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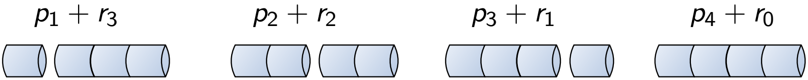

Recursive Top-down Implementation

Recall:

$$r_n = \max_{1 \leq i \leq n} (p_i + r_{n-i}) \text{ and } r_0 = 0 \ .$$

Direct Implementation:

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Require: Integer n, Array p of length n with prices
  if n = 0 then
    return 0
  q ← −∞
  for i = 1 ... n do
    q ← max{q, p[i] + CUT-POLE(p, n − i)}
  return q
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Algorithm CUT-POLE(p, n)

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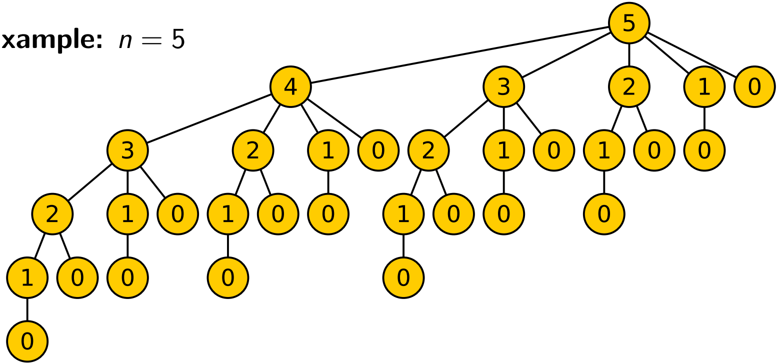
Algorithm $\textsc{Cut-Pole}(p, \ n)$

How efficient is this algorithm?

Example: $n = 5$



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Recursion Tree for Cut-Pole

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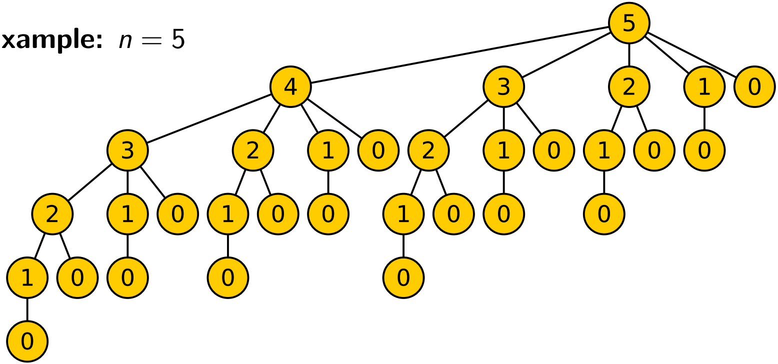

Number Recursive Calls: $T(n)$

$$T(n) = 1 + \sum_{j=0}^{n-1} T(j) \text{ and } T(0) = 1$$

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This implies $T(i) = 2^i$.

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What can we do better?

- Observe: We compute solutions to subproblems many times
- Avoid this by storing solutions to subproblems in a table!
- This is a key feature of dynamic programming

Implementing the Dynamic Programming Approach

Top-down with memoization

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- Fill table T from smallest to largest index

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- When computing r_i, store r_i in a table T (of size n)
- Before computing r_i again, check in T whether r_i has previously been computed

Bottom-up

- Fill table T from smallest to largest index
- No recursive calls are needed for this

Top-down Approach

Require: Integer n, Array p of length n with prices
  Let $r[0 \dots n]$ be a new array
  for $i = 0 \dots n$ **do**
    $r[i] \leftarrow -\infty$
  return MEMOIZED-CUT-POLE-AUX(p, n, r)

Algorithm MEMOIZED-CUT-POLE(p, n)

- Prepare a table r of size n
- Initialize all elements of r with $-\infty$
- Actual work is done in MEMOIZED-CUT-POLE-AUX, table r is passed on to MEMOIZED-CUT-POLE-AUX

Top-down Approach (2)

Require: Integer n, array p of length n with prices, array r of revenues

```
if r[n] ≥ 0 then
    return r[n]
if n = 0 then
    q ← 0
else
    q ← −∞
    for i = 1 . . . n do
        q ← max{q, p[i] + MEMOIZED-CUT-POLE-AUX(p, n − i, r)}
r[n] ← q
return q
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Algorithm MEMOIZED-CUT-POLE-AUX(p, n, r)

Observe: If $r[n] \geq 0$ then $r[n]$ has been computed previously

Bottom-up Approach

Require: Integer n, array p of length n with prices
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  for $j = 1 \dots n$ **do**
    $q \leftarrow -\infty$
    for $i = 1 \dots j$ **do**
      $q \leftarrow \max\{q, p[i] + r[j - i]\}$
    $r[j] \leftarrow q$
  return $r[n]$

Algorithm Bottom-Up-Cut-Pole(p, n)

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Algorithm BOTTOM-UP-CUT-POLE(p, n)

Runtime: Two nested for-loops

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    $q \leftarrow -\infty$
    for $i = 1 \dots j$ **do**
      $q \leftarrow \max\{q, p[i] + r[j-i]\}$
    $r[j] \leftarrow q$
  return $r[n]$

Algorithm $\textsc{Bottom-Up-Cut-Pole}(p,\ n)$

Runtime: Two nested for-loops

$$\sum_{j=1}^{n} \sum_{i=1}^{j} O(1) = O(1) \sum_{j=1}^{n} \sum_{i=1}^{j} 1 = O(1) \sum_{j=1}^{n} j = O(1) \frac{n(n+1)}{2} = O(n^2) \ .$$

Conclusion

Runtime of Top-down Approach $O(n^2)$

(please think about this!)

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Dynamic Programming

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- Top-down and bottom-up approaches have the same runtime