Big-O Notation

COMS10018 - Algorithms

Dr Christian Konrad

Definition: O-notation ("Big O")

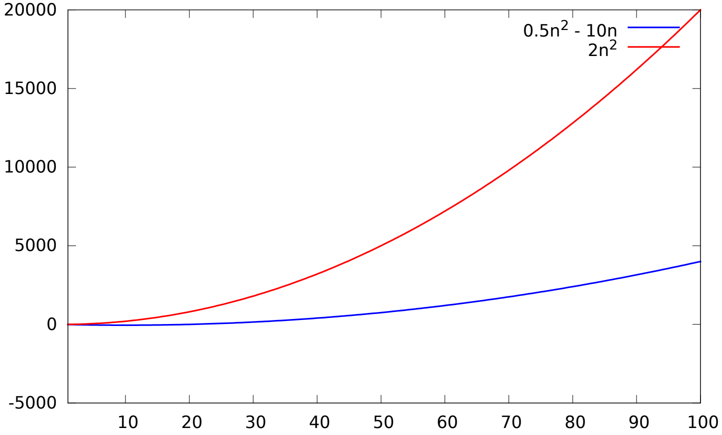
Let $g(n)$ be a function. Then $O(g(n))$ is the set of functions:

$$O(g(n)) = \{f(n) : \text{There exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$$

Meaning: $f(n) \in O(g(n))$: "g grows asymptotically at least as fast as f up to constants"

O-Notation: Example

Example: $f(n) = \frac{1}{2}n^2 - 10n$ and $g(n) = 2n^2$



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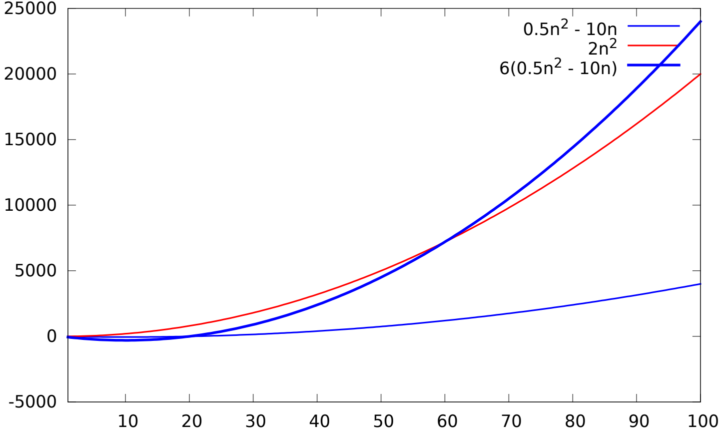

Then: $g(n) \in O(f(n))$, since $6f(n) \geq g(n)$, for every $n \geq n_0 = 60$

More Examples/Exercises

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Exercises:

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$$\begin{aligned} 0.5n &\leq cn/\log n \\ \log n &\leq 2c \\ n &\leq 2^{2c} \text{, a contradiction,} \end{aligned}$$

since this does not hold for every $n > 2^{2c}$.

Recipes

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Find constants c, n_0 as in the statement of the definition of Big-O, i.e., such that $f(n) \leq c \cdot g(n)$, for all $n \geq n_0$

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Proving that $f \notin O(g)$:

Proof by contradiction: Assume that constants c, n_0 exist as in the statement of the definition of Big-O and derive a contradiction

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Lemma (Sum of Two Functions)

Suppose that $f, g \in O(h)$*. Then:* $f + g \in O(h)$.

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Since $f \in O(h)$ there exist constants c, n_0 such that

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$$f(n) + g(n) \leq ch(n) + c'h(n) = C \cdot h(n) \text{ for every } n \geq N_0 \ . \quad \square$$

Further Properties

Lemma (Polynomials)

Let $f(n) = c_0 + c_1 n + c_2 n^2 + c_3 n^3 + \cdots + c_k n^k$*, for some integer* k *that is independent of* n*. Then:* $f(n) \in O(n^k)$.

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Proof: Apply statement on last slide $k = O(1)$ times $\square$

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Application of statement on last slide n times! (only allowed to apply statement $O(1)$ times!)

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- Loops: (repetition of instructions)

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